MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. A. C. D. CROMMELIN, PRESIDENT, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Ernest Reginald Adams, B.Sc., 124 Croxted Road, West Dulwich, London, S.E. 21 ;

T. Arnold Brown, M.A., B.Sc., Professor of Mathematics, University College of the South-West of England, Exeter ;

Donald Cameron-Swan, F.R.P.S., F.S.A.Scot., Captain, late R.A.F., Strathmore, Kalk Bay, C.P., South Africa ;

Alfred George Course, Captain, Merchant Navy, Assistant Dock-master to Port of London Authority, 5 The Gardens, Tilbury, Essex ;

Thomas George Cowling, B.A., 52 Lyndhurst Drive, Leyton, London, E. 10 ;

George A. Davis, Jr., 936 Ellicott Square, Buffalo, New York, U.S.A. ;

Hugh Dawson, The Southern Cross Assurance Co., Ltd., Wellington, New Zealand ;

Vincenzo Consolato Antonino Ferraro, 23 Princes Park Avenue, Golders Green, London, N.W. 11 ;

Rowdon Marrian Fry, M.R.C.S., L.R.C.P., 73 East Sheen Avenue, London, S.W. 14 ;

Percy Joseph Harris, B.Sc., Hillcroft, Shrewsbury Avenue, Market Harborough, Leicestershire ;

Eric Ludlow Hawke, M.A., F.R.Met.Soc., Caenwood, The Valley Road, Rickmansworth, Herts ;

Christian Victor Charles Herbert, B.A., 53 Sloane Gardens, London, S.W. 1 ;

William Moffat Inverarity, M.A., B.Sc., 11 Ravenshall Road, Glasgow, S. 1 ;
Derek Ainslie Jackson, Clarendon Laboratory, University Museum, Oxford ;
Floris Jansen, Instituto Geográfico Militar, Gral. Paz 1314, Buenos Aires, República Argentina ;
L. W. P. Lewis, Solva, Mawgan Porth, St. Columb, Cornwall ;
Cyrus Augustus Macfarlane, Ardath, Venner Road, Annerley, Brisbane, Australia ;
Donald Glenoe McIntyre, Civil Service Club, Cape Town, S. Africa ;
Rev. William Isaac H. Marr, M.A., B.Sc., St. James's Manse, Kilmacolm, Renfrewshire ;
V. V. Narliker, B.A., Fitzwilliam House, Cambridge ;
Frank Potter, 8 Ranelagh Gardens, Hurlingham, London, S.W. 6 ;
Donald Harry Sadler, B.A., 6 Wilmore Road, Forest Hill, London, S.E. 23 ;
Russell Sullivan, B.A., 1431 North Meridian Street, Indianapolis, Indiana, U.S.A. ;
Noel Thomas, Heather Mount, Borough Green, Kent ;
Arthur Wilfred Tucker, M.A., B.Sc., Training College, Dunedin, New Zealand ;
Rev. Walter Walters, The Rectory, Scottsdale, Tasmania, Australia ; and
Francis John Welsh Whipple, Sc.D., F.Inst.P., Director of the Kew Observatory, 6 Addison Road, Chiswick, London, W. 4.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Hubert Slouka, Astronomical Institute of the University Charles IV., Prague (proposed by W. M. H. Greaves) ; and
George Kraal Thornhill, Acting Surveyor-General, Colombo, Ceylon (proposed by A. J. Bamford).

Fifty-nine presents were announced as having been received since the last meeting.

The President announced that the Gold Medal of the Society had been awarded to Professor W. de Sitter, for his theoretical investigations on the orbits of the satellites of Jupiter and his contributions to the theory of relativity ; and that a Jackson-Gwilt Medal and gift had been awarded to Mr. Clyde W. Tombaugh, of the Lowell Observatory, for his discovery of the extra-Neptunian planet Pluto.

The President further announced that Professor de Sitter had been invited to deliver the George Darwin Lecture for 1931.

A New Bright-hydrogen-line Star of Spectral Type B8 in Lacerta.

By William J. S. Lockyer, M.A., Ph.D., Major (late R.A.F.).

In an inquiry on the spectra of B-type stars which exhibit bright hydrogen lines a search has recently been made among the numerous negatives taken at the Norman Lockyer Observatory with the 12-inch prismatic camera embodying a prism of 20° angle, to discover any spectra of this type which contained or seemed likely to contain bright hydrogen lines.

During this search a star in Lacerta, namely, H.D. 216057 or H.R. 8682 (1900 R.A. $22^h\ 44^m{\cdot}6$, Dec+53° 53′, Mag. 6·08, Sp. Type B8) was found to have this peculiarity.

The line $H\beta$ was first suspected to be bright on a rather underexposed negative taken on 1928 August 17, but on a subsequent and denser negative secured on October 1 of the same year the bright line in the centre of the broad $H\beta$ absorption band is strong and quite clearly pronounced.

In order to secure, if possible, better negatives, the star has recently been photographed again, on 1930 November 6 and December 9.

Both these photographs show $H\beta$ as a broad absorption band with a strong bright line down its centre: whether this line is single or a close double cannot be detected owing to the small dispersion employed. In the photograph taken on the latter date $H\gamma$ is suspected of having a similar bright line in the centre of the broad absorption.

The only other lines visible in the spectrum are those of helium, which are broad and nebulous as in many other stars of this spectral type. The line of ionised magnesium at λ 4481 and that of ionised calcium [K] seem to be absent.

To determine whether these bright lines had been recorded before, search was made among all the available publications in which this B type of spectrum had been described.

In the Revised Harvard Photometry * no reference is made to any bright lines having been recorded.

Merrill,† whose observing programme at the Lick Observatory in 1911 and 1912 with the 36-inch refractor included both B stars with known bright lines and others with possible bright lines, does not mention this star.

Moore,‡ in his "Third Catalogue of Spectroscopic Binary Stars," does not include it in any of his tables.

In the Henry Draper Catalogue § no bright lines are mentioned.

Merrill, Humason, and Burwell,‖ in their investigation entitled "Discovery and Observations of Stars of Class Be," seem to have missed this star in their survey.

* *H.C.O. Ann.*, **50**, 191, 1908.
† *L.O.B.*, No. 237, 1913.
‡ *L.O.B.*, No. 355, 1924.
§ *H.C.O. Ann.*, **99**, 169, 1924.
‖ *A.J.*, **61**, 1925.

The very comprehensive list of "Radial Velocities of 368 Helium Stars," published by Frost, Barrett, and Struve * in 1926, omits this star.

The above references indicate that this is the first occasion of bright lines having been recorded in the spectrum of this star. It would be interesting if those observers who have previously photographed its spectrum would re-examine their negatives, especially at $H\beta$ and $H\gamma$, to find out whether the bright lines are really absent or perhaps only very faint.

In any case it would seem that the bright lines are variable, considering the intensity of that in $H\beta$ at the present epoch.

According to the international notation now adopted, the spectrum of this star must now be written as B8neβ.

Finally, as it is well known that stars in the spectral classes Boe to B5e tend strongly to cluster towards the galactic equator, this star, though it is of spectral class B8e, seems to follow the same law, its galactic latitude being about $-4°$.

The Shift towards Red of the Calcium, Aluminium, and Iron Lines in the Solar Spectrum. By J. Evershed, F.R.S.

Measures of the shift towards red of the calcium lines H and K in the prominences give a value greatly in excess of the relativity shift. The mean shift resulting from my measures of about 380 prominence spectra at various points on the limb of the Sun is + 0·015 A., or 0·007 in excess of the Einstein shift. It is of interest to compare this result with the shifts of the same lines in the higher chromosphere, and of other lines representing different levels in the reversing layer.

During the years 1929–30 a considerable number of limb spectra have been secured at Ewhurst, as well as spectra of the centre of the Sun's disc. I have confined attention entirely to the region of the H and K lines, and all the spectra obtained include about 100 angstroms in this region. They are on a scale of approximately 1 mm. to the angstrom at the end towards red, increasing to 1·1 mm. at the more refrangible end, and the scale between these limits is taken as a linear function of the wave-length.

The instrument used is the large auto-collimating prism spectroheliograph used alternatively as a spectrograph. The collimator-camera has a focal length of about 16 feet, and there are three large prisms of 6-inch effective aperture, two of 45° angle, and one reflecting prism of 30°. For this special work, where exposure times are not, as with prominence spectra, an important consideration, it is found advantageous to substitute a 4-inch achromatic lens for the 6-inch simple lens used for the spectroheliograph and for prominences. The

* *A.J.*, **64**, 1926 July.

smaller lens sends the beam of light through the apical portion of the large prisms, in this way avoiding the great thickness of glass of the basal part, where absorption of ultra-violet light is serious. The plate-holder with this lens is fixed very nearly normal to the axis of the lens, which is an advantage compared with the simple lens requiring an inclination of 13° and very careful adjustment of the spectrum on the plate to ensure perfect focus. The definition of the arc and solar lines is perfect, and of course there is no trouble from ghosts, as with grating spectra.

The spectra of calcium, aluminium, and iron are obtained from an arc of the Pfund type burning in air. The pole-pieces are of mild steel, the lower one of sufficient thickness to prevent its getting too hot and burning away. The upper end is hollowed out, and in this is placed a small piece of carbon of a brand which happens to give very narrow, well-defined calcium and aluminium lines. The arc is run at 5 amperes and 100 volts. The arc spectra are impressed on both sides of the narrow strips of solar spectra, and for the chromosphere the east and west limbs are photographed side by side on the same plate, with the comparison spectrum outside, the total width being well within the field of view of the measuring micrometer. Exposures are made alternately on Sun and arc and Sun again. The stability of the apparatus and the unvarying temperature of the prisms and lens, which are enclosed in a small recess 10 feet below ground, is ample guarantee against instrumental shift.

The measured shifts are of course freed from the effects due to the Earth's rotation and orbital motion in the direction of the Sun, and for the limb shifts the Sun's rotation is also eliminated, by taking half the difference of shift west−east. As is well known, the shifts of the solar lines vary according to the position on the Sun's disc whence the light is derived, and for most lines the shift is greater at the limb than at the centre of the disc. In *Kodaikanal Observatory Bulletin*, No. 49, 1916, it was shown that this relative shift between centre and limb is large for certain lines of medium intensity in the red region, and for weak lines in the violet region, but is very small for certain strong lines in the violet. The measures at intermediate points between centre and limb showed that the wave-length began to change quite near the centre, increasing slowly at first, then more rapidly as the limb was approached. The increase is in fact for some lines roughly proportional to the versed sine of the central distance, a relation that seems to suggest a Doppler effect due to a radial outflow of the gases giving the particular lines studied.

Other lines, however, showed a considerable departure from this relation. Any radial movement of the gases upwards or downwards would of course have its greatest effect at the centre of the disc, the component of motion in the line of sight vanishing at the limb, where the relativity effect ought to appear undisturbed by any systematic motion, unless this were directed away from the Earth.

In the following tables I give the observed shifts of the Ca, Al, and Fe lines at the Sun's centre and limb.

TABLE I.

Centre of Sun—Arc in Air, in A/1000.

λ Revised Rowland.	Intensity.	Kodaikanal, 1912–1922.		Ewhurst, 1929–1930.		Revised Rowland minus 1922 Arc Standards.
		Δλ	No.	Δλ	No.	Δλ.
3885·521	4	+ 4	4	..	..	..
3886·296	15	12	8	..	..	+11
3887·061	7	9	8	..	..	9
3888·526	5	8	8	..	..	8
3895·669	7	12	20	12	4	10
3899·721	8	13	20	14	8	11
3902·958	10	10	20	12	14	9
3906·492	10	10	20	12	16	10
3920·271	10	10	29	12	19	10
3922·925	12	10	29	10	19	10
3925·653	5	3	21	4	8	..
3927·935	8	11	29	12	20	12
3930·310	8	11	29	11	20	10
3933·684 K	1000	..	..	18	13	20
3935·828	2	10	21	12	7	10
3940·892	5	6	16	6	8	8
3944·018 Al	15	6	6	5	8	..
3949·963	5	6	22	7	13	..
3956·465	4	..	..	2	11	4
3956·688	6	4	25	6	13	6
3961·537 Al	20	8	8	7	8	..
3966·075	3	5	19	6	13	8
3968·494 H	700	..	..	20	12	23
3969·270	10	9	18	12	13	8
3971·334	5	..	..	8	11	7
3977·752	6	4	12	6	13	6

The shifts of H and K in the last column are the differences between my values for the lines in the arc and the *Revision of Rowland's Table.*

I give first in Table I. the shifts at the Sun's centre expressed in A/1000. In the years 1912–22 I made a large number of measures by the positive-on-negative method of both grating and prism spectra, and the mean values obtained, which have not hitherto been published, are given in the third column, with the number of measures in the fourth column. The recent measures made by the ordinary bisection method, with the number of measures on which the values depend, are given in the next two columns, and the last column gives the Mount Wilson values; these are obtained from the differences between the Revised

Rowland's Table of Solar Wave-lengths and the Secondary and Tertiary Iron Arc Standards of 1922, on which the Revision has been founded. The general agreement in the three columns of $\Delta\lambda$ indicates the order of reliability of these values.

The Ewhurst values tend to be larger than Kodaikanal: this may be the result of employing a smaller dispersion, and not measuring by the positive-on-negative method, resulting in the actual intervals measured being from three to four times smaller in the Ewhurst determinations.

TABLE II.

Limb of Sun—Arc in Air, in A/1000.

λ Revised Rowland.	Intensity.	Kodaikanal, 1921–1923.		Ewhurst, 1929–1930.	
		Δλ.	No.	Δλ.	No.
3885·521	4	**10**	23	..	..
3886·296	15	**13**	27	..	..
3887·061	7	**13**	27	..	..
3888·526	5	**10**	27	..	..
3895·669	7	**15**	27	**10**	5
3899·721	8	**15**	27	**15**	10
3902·958	10	**11**	27	**12**	11
3906·492	10	**15**	27	**15**	16
3920·271	10	**14**	43	**14**	35
3922·925	12	**15**	43	**13**	36
3925·653	5	**11**	17	**13**	4
3927·935	8	**13**	40	**14**	36
3930·310	8	**15**	35	**13**	37
3933·684 K	1000	..	..	**13**	36
3935·828	2	**16**	20	**16**	8
3940·892	5	**16**	17	**11**	12
3944·018 Al	15	**5**	21	**5**	25
3949·963	5	**14**	25	**11**	19
3956·465	4	..	..	**9**	21
3956·688	6	**13**	27	**11**	29
3961·537 Al	20	**6**	23	**6**	28
3966·075	3	**13**	25	**11**	24
3968·494 H	700	..	..	**13**	39
3969·270	10	**11**	25	**15**	34
3971·334	5	..	..	**11**	16
3977·752	6	**10**	12	**11**	27

Table II. gives similarly my values of the limb shifts made at Kodaikanal and at Ewhurst, with the number of measures on which

each value depends. Here again the agreement in the two sets is satisfactory, notwithstanding the differences in the instruments used and the methods of measurement.

It is doubtful, from my experience, if the shifts at centre or limb vary systematically with the sunspot cycle, but changes do frequently occur, owing possibly to local currents in the Sun; and the limb spectra may give values differing widely from the mean owing to irregularities in the rotation shift, which may be unequal at opposite ends of the Sun's diameter. For these reasons only mean values from a considerable number of plates, rejecting none, can give trustworthy results. In one recent plate of the centre of the Sun, strong lines show a mean defect of ·002 A., and the weak lines a mean defect of ·006 A. This may be interpreted as an upward current affecting mainly the lower levels in the reversing layer.

The twenty-two iron lines measured in this region are all symmetrical lines in the arc, and are not appreciably subject to pole-effect. One-half of these are strong lines with a mean intensity on Rowland's scale of 9·5 in the Sun. The remaining eleven lines are much weaker, having a mean intensity of 4·5. For these groups there is an obvious relation between the intensity of a line and the amount of the shift at the Sun's centre, whilst at the limb all the Fe lines give nearly the same shift.

The relation between the shifts at centre and limb may best be seen by analysing Table I. into the different intensity groups. The strongest lines, H and K, or rather their narrow central cores, H_3 and K_3, give a very large shift at the centre, which actually decreases to the usual value for the other lines at the limb. The next strongest lines are those of aluminium: here there is no appreciable change between centre and limb, as I have found also to be the case with the D lines of sodium (1). But the observed shift is only half the usual value. This is due to the fact that the aluminium lines in the arc in air are subject to a large pressure shift, and are also subject to pole-effect: they occur in the carbon arc as very sharp lines, apparently stable in position. The next in order of intensity are the eleven strong iron lines: these show a large centre—arc shift increasing only a small amount at the limb. Finally, the weaker iron lines give small centre shifts which increase to nearly the same values as the shifts of the strong lines at the limb.

These results are readily appreciated by referring to Tables III., IV., and V., where I give the mean values of the shifts of each group corrected for the mean pressure effect of the arc in air. The corrections are taken from Adams and Gale's measures of arc spectra for the iron lines at 8 atmospheres (2), and Humphreys's measures at 42 atmospheres for the calcium and aluminium lines (3). The pressure effect for 1 atmosphere is assumed to be 1/8th of Adams's values and 1/42nd of Humphreys's. Humphreys's and Adams's measures for iron are in good agreement line by line, although Adams's yield slightly larger values than Humphreys's. The mean difference is, however, only ·0005 A.

TABLE III.

Mean Shifts of Calcium and Aluminium Lines in A/1000.

Calcium.

	Δλ ⊙.	Δλ limb.
K	+18	+13·4
H	+20	+13·4
Means	+·0190 A.	+·0134 A.
1 A^t pressure	+ 17	+ 17
Sun—vacuum arc	+·0207 A.	+·0151 A.

Aluminium.

	Δλ ⊙.	Δλ Limb.
3944	+5	+5
3961	+7	+6
Means	+·0060 A.	+·0055 A.
1 A^t pressure	+ 44	+ 44
Sun—vacuum arc	+·0104 A.	+·0099 A.

TABLE IV.

Mean Shifts of 11 Strong Fe Lines in A/1000.

λ.	Intensity.	Δλ ⊙.	Δλ Limb.
3886·296	15	12	13
3887·061	7	9	13
3895·669	7	12	14
3899·721	8	13	15
3902·958	10	11	11
3906·492	10	11	15
3920·271	10	11	14
3922·925	12	10	14
3927·935	8	11	13
3930·310	8	11	14
3969·270	10	11	15
Means	9·5	+·0111 A.	+·0137 A.
1 A^t pressure		+ 18	+ 18
Sun—vacuum arc		+·0129 A.	+·0155 A.

TABLE V.

Mean Shifts of 11 *Weak Fe Lines in A*/1000

λ.	Intensity.	Δλ ⊙.	Δλ Limb.
3885·521	4	4	10
3888·526	5	8	10
3925·653	5	3	11
3935·828	2	11	16
3940·892	5	6	14
3949·963	5	6	13
3956·465	4	2	9
3956·688	6	6	12
3966·075	3	6	12
3971·334	5	8	11
3977·752	6	6	11
Means	4·5	+·0060 A.	+·0117 A.
1 At pressure		+ 21	+ 21
Sun—vacuum arc		+·0081 A.	+·0138 A.

In the case of aluminium, owing to pole effect and consequent uncertainty of the pressure correction, there must remain doubt as to the absolute amount of the shift. The only certain result for this element is the absence of appreciable relative shift between centre and limb.

In interpreting the shifts at the centre of the Sun, the question of level is of great importance. It is quite certain that the central narrow absorption lines, H_3 and K_3, represent the upper chromosphere. Next in order of height we may put the sodium and aluminium lines: the evidence given by the lengths of arcs in eclipse spectra shows that these elements occupy a high level in the reversing layer, whilst the central nuclei of the strong winged lines of iron probably represent a slightly lower level. The effective level of absorption of the weak iron lines is certainly lower than that of the strong iron lines. This is also indicated by eclipse spectra, and their motion shifts in sunspot spectra also point to a difference of level.

It may appear difficult to understand why weak and strong lines of the same element should represent different regions in the Sun, when under laboratory conditions they occur together in the same small masses of gas. But the explanation is quite simple in the case of the two groups I have studied, for of the strong lines, which are winged, only the central core is measured, and this certainly represents a higher level than the line taken as a whole, which may represent the same effective region of absorption as the weak narrow lines of the same element. The calcium lines are an extreme case: here the central absorption line K_3 represents the upper chromosphere, whilst

the band as a whole represents absorption at all levels. This explanation of course does not apply to narrow, sharply defined lines of differing intensities, and it would seem that lines of this character belonging to the same multiplet group must represent the same level, and should be equally affected by changes of wave-length. But narrow lines of different elements, or of different atomic levels in the same element, might show this relation of shift to intensity due to a difference of level in the Sun. The extreme tenuity of the solar gases is such that a depth of many hundreds of kilometres is necessary to produce absorption, and weak lines will require a much greater depth than strong lines, consequently the effective region of absorption for these would be nearer the photosphere.

In the present case it is clear that we have four distinct levels represented by the lines that I have measured, and the shifts observed appear to be consistent with the relativity effect modified by Doppler effects, as advocated by St. John. According to him the lowest levels, represented by lines of intensities below 5, yield shifts in the violet region which are less than the relativity effect, this being the result of a radial outflow at low levels partly counteracting relativity. At the higher level of about 520 km. above the photosphere this motion appears to cease, and lines of mean intensity 6, representing this level, give exactly the Einstein shift. At levels upwards of 1000 km., represented by the strong lines that I have measured, the motion is slightly downward, the shift slightly exceeding the theoretical.

My results do not include the lines of lowest level, but the weak iron lines of mean intensity 4·5 give a value very slightly under the theoretical if the pressure correction is not seriously wrong. All these lines are of class *b*, except one of class *a*, and the correction is likely to be close to the value given in the table, which is the mean value for three of the lines whose pressure shifts are known.

The strong lines give an excess over relativity equivalent to a descending motion of ·35 km./sec., whilst for H_3 and K_3 the excess is equal to ·94 km./sec. descent. On the whole, but excluding the high-level calcium, the average shift of all the other lines studied, including aluminium, is very close to the Einstein effect.

Notwithstanding this general agreement, which occurs also in other spectral regions, for the centre of the Sun, Messrs. Burns and Meggers consider that owing to the many factors involved it is unwise to form a judgment at present concerning the cause of the red shifts, which they suggest might be due to the majority of the solar lines being slightly unsymmetrical (4). This would result in the stronger absorption lines being more shifted than the weaker. But it would be necessary to assume also that all the lines giving red shifts were alike shaded towards red and better defined on the violet side in order that the increase of shift with the increase of absorption should always be towards red. My observations, especially those made with the positive-on-negative micrometer, seem to rule out this suggestion entirely, for if a carefully made positive is superposed on a negative of the solar spectrum a want of symmetry in a line is at once revealed in the micrometer micro-

scope when the positive image of a line is reversed on the negative image. Instead of a perfectly uniform shading, an obvious darkening on one side is seen. This is a very delicate test, and it has always indicated perfect symmetry for isolated solar lines, such as those I select for measurement. In fact, want of symmetry is only shown in the case of lines having faint close companions. It is true that some of the stronger winged lines may be slightly unsymmetrical in the distribution of the shading, as noted by d'Azambuja (5). But measures of position are made on the central core of these lines, representing the less dense gas at comparatively high levels. For such lines my measures have been made mostly on plates that have been overexposed for the weaker lines, and in these spectra the central cores stand out clearly and give no suggestion of asymmetry in the sense required.

The Limb Shifts.—Considering now the shifts at the limb, shown in the above tables, the most striking facts are the uniformity of the shift for the mean of every group, except aluminium, which is uncertain, and the agreement with the prominence result. If we subtract the relativity shift from the means and include the prominence result, we obtain the following :—

Group.	Limb Shift.	Relativity Shift.	Residual Shift.
H and K in prominences	+·0151	+·0083	+·0068
H and K in chromosphere	+·0151	+·0083	+·0068
Strong Fe lines	+·0155	+·0083	+·0072
Weak Fe lines	+·0138	+·0083	+·0055

The shifts of the weak iron lines are probably underestimated, because the spectra are obtained with light derived from a region from 5″ to 10″ within the limb. It is impossible to get a pure spectrum of the limb itself unaffected by admixture of general sunlight, except during a large eclipse of the Sun. For these lines there is a rapid increase of shift as the limb is approached, whereas the strong lines show a very small increment between centre and limb, and probably do not increase appreciably in wave-length near to the limb. Eclipse spectra might show complete equality among all intensity groups.

As already mentioned, my results from a very limited region of spectrum are in substantial agreement with St. John's based on a much greater mass of material and extending through a great range of wave-length (6). The point of greatest difference between us is in the amount of the limb shift, especially that given by the stronger lines. St. John's measures do not include many of the lines in the H and K region, and the only lines we have measured in common are the following :—

3885·521	3888·526	3902·958
3886·296	3899·721	3906·492

The mean values for these, limb – arc (reduced to vacuum), are as follows :—

St. John.	Evershed.
+ ·0107 A.	+ ·0142 A.

St. John appears to have applied the same pressure coefficients as I have to reduce the arc to vacuum. I can only say that my results depend on a large number of measures of plates obtained with different apparatus, and the separate determinations are consistent, as is seen in Table II. At the centre of the Sun we have measured many lines in common, and here the results are in better agreement, as Table I. indicates.

There remains to be explained the mysterious excess of shift at the limb, amounting, according to my measures, to about ·007 A., or, expressed in motion, a little over ·5 km./sec. This cannot be explained by Compton scattering due to the long optical path from the limb, for exactly the same excess is shown by the prominence spectra, where the light does not traverse any portion of the chromosphere and the lines are not widened as they are in limb spectra. If we suppose it to be a Doppler effect, we have to assume a recession at all points on the limb directed at all times away from the Earth. This objectionable Earth effect in combination with St. John's radial movements may possibly explain the way in which the shift varies between centre and limb, both for low-level and high-level lines.

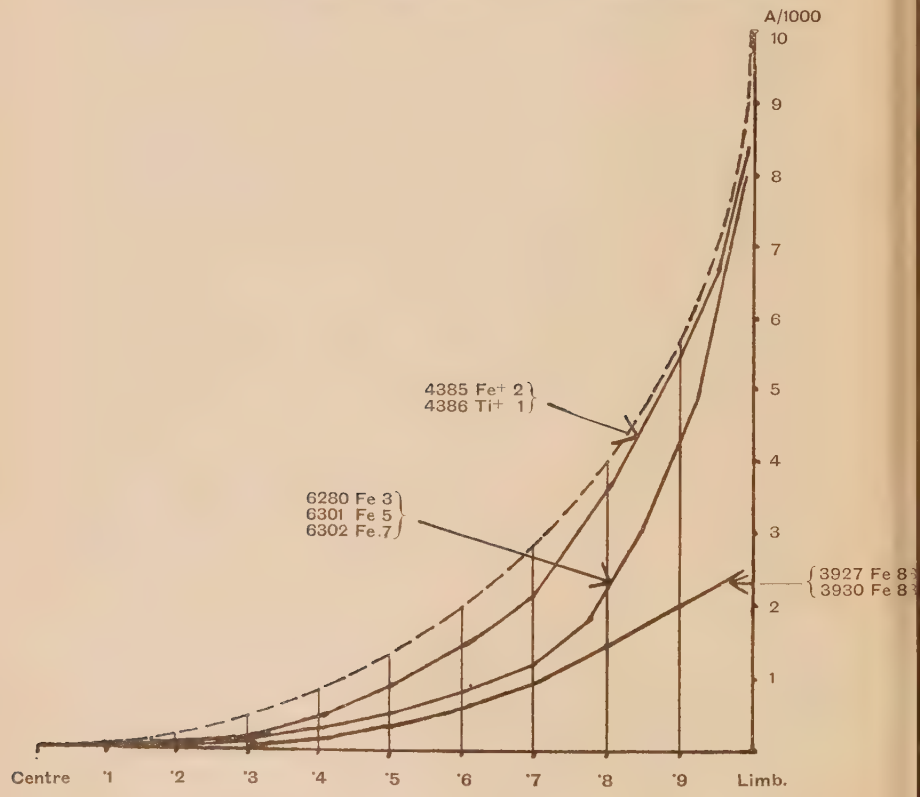
The diagram on p. 270 shows the variation between centre and limb for three groups of lines representing different levels in the reversing layer. I have taken this from the *Kodaikanal Observatory Bulletin*, No. 49, 1916, but I have averaged the values in the different intensity groups which are given separately in the bulletin. I have also added in broken lines a curve of which the ordinates are the versed sines of the central distances.

It is remarkable that among the lines I measured, those of intensity 1 and 2, due to ionised iron and titanium, and representing low levels, give values approximating quite closely to the curve, which represents cosines when measured downwards from the top of the diagram. The three lines in the red of average intensity 5 give mean values which show a considerable departure from this relation ; and the two strong Fe lines of intensity 8, representing the highest region, show a much greater departure. Assuming that the residual shifts are Doppler effects, we must suppose that the highest layers come under the influence of Earth-repulsion, which also affects quite strongly the high-level chromosphere and prominences, whilst the low-level gases are subject to the outward radial movement, as suggested by St. John.

More observational material is of course needed to determine as accurately as possible the law of increase of wave-length between centre and limb for different classes of lines. The only measures that have been made since my own are by Freundlich, who finds for certain multiplets of iron that the increase of wave-length begins about 45° from the centre of the disc and does not follow the cosine curve : he therefore considers " that it is as impossible to explain the effect by radial streaming as by simple Compton scattering " (7).

The evidence for an outward motion for low-level gases and inward for high-level calcium, with zero movement at about the level of sodium and aluminium, suggests a striking analogy with the radial motion in

sunspots, where the low-level gases are moving outwards parallel with the Sun's surface, and the high-level calcium is moving inwards, the zone of no motion being about the level of sodium absorption. In



Change of Wave-length between Centre and Limb.

neither case is there much evidence of a return movement of the same elements. If we accept the shifts in spot spectra as Doppler effects, due to outward and inward motion, there would seem little reason to doubt that the residual shifts at centre and limb are also evidences of motion.

References.

(1) *Observatory*, **46**, 303 (1923).
(2) *Astrophysical Journal*, **35**, 17 (1912).
(3) *Astrophysical Journal*, **26**, 23 (1907).
(4) *Publications of the Allegheny Observatory*, **6**, 123: and *Journal of the Optical Society of America*, **20**, 212 (1930).
(5) *Annales de l'Observatoire de Paris*, Section d'Astrophysique, **8**, 68 (1930).
(6) *Contributions from the Mount Wilson Observatory*, 348; and *Astrophysical Journal*, 67 (1928).
(7) Freundlich, Brunn, and Brück, *Zeitschrift für Astrophysik*, **1**, 43 (1930).

Mean Areas and Heliographic Latitudes of Sunspots in the Year 1929.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those in the *Monthly Notices*, **90**, 303, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at the Kodaikanal Observatory. Photographs for June 30 and November 6 were kindly lent for measurement by the Yerkes Observatory.

Table I. gives the mean daily areas of umbræ, whole spots, and faculæ for each synodic rotation of the Sun included in the year 1929, whilst the mean values for each year as a whole from 1923 to 1929 are given in Table II.

Table III. gives for each rotation in the year 1929 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for both hemispheres, northern and southern. The mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots are also tabulated. The mean values for the years 1923 to 1929 are given in Table IV. Tables II. and IV. are in continuation of similar tables given in the *Monthly Notices*: for the years 1874 to 1888, **49**, 381 and 382; 1889 to 1902, **63**, 465 and 466; 1901 to 1914, **76**, 402 and 403; 1913 to 1924, **85**, 1007 and 1008.

The rotations in Tables I. and III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node of the Sun's equator on the ecliptic at mean noon on 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

The principal features of the record for the year 1929 were as follows:—

1. A slight decrease in the mean daily areas of spots and an almost identical amount of faculæ as compared with the mean values for 1928.

2. Practically the same number of groups of spots appeared as in 1928. Groups of two days' duration and more showed a slight decrease in number which was compensated by an increase in the number of spots seen on one day only.

3. The northern and southern hemispheres were about equally active.

4. The mean latitude of spots ($10\frac{1}{2}^\circ$) was 3° nearer the equator than in 1928. Latitudes ranged from 1° to 25° (there was a one-day spot in latitude 35° on February 27).

5. On no days were spots and faculæ absent from the Sun's disc.

6. The number of groups lasting for two days or more was 321; of these 168 were in the northern hemisphere and 153 in the southern. Besides these, 148 very small groups (mostly consisting of one or two

tiny spots) were measured on one day only; 65 of these were in the northern hemisphere and 83 in the southern.

7. It will be seen from Table I. that during the year 1929 there were considerable fluctuations in sunspot activity. During rotation No. 1016, the mean daily area corrected for foreshortening dropped to 273, whilst in rotation 1019 the corresponding value was 3082—a mean area surpassed only once during the present cycle, in rotation 966, December 1925. The faculæ, however, were more constant from rotation to rotation as is frequently the case.

An inspection of Table II. shows the indefinite nature of the maximum of the present cycle of activity which appears now to be declining rapidly.

Table I.

No. of Rotation.	Rotation commenced G.C.T.	No. of Days on which Sun Photographed.	Mean of Daily Areas. Projected.* Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening.† Umbræ.	Whole Spots.	Faculæ.
1007	Dec. 25·85	27	241	1437	1753	171	1006	2148
1008	Jan. 22·19	28	250	1247	1916	174	875	2255
1009	Feb. 18·53	27	303	1562	1879	231	1187	2270
1010	Mar. 17·86	27	212	1050	1622	157	786	1942
1011	Apr. 14·14	28	262	1314	2175	198	999	2549
1012	May 11·39	27	187	915	2284	147	739	2625
1013	June 7·60	27	373	1800	2449	274	1342	2848
1014	July 4·80	27	287	1362	2494	208	976	2837
1015	Aug. 1·01	27	253	1169	2505	181	836	2858
1016	Aug. 28·24	28	78	348	2331	60	273	2681
1017	Sept. 24·50	27	309	1641	1850	221	1172	2176
1018	Oct. 21·79	27	496	2569	2043	362	1869	2372
1019	Nov. 18·10	28	812	4171	2540	619	3082	3042

Table II.

Year.	No. of Days. Photographed.	Without Spots.	Mean Daily Areas. Projected.* Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening.† Umbræ.	Whole Spots.	Faculæ.
1923	365	171	15	74	189	10	55	222
1924	366	97	62	361	498	46	276	575
1925	365	19	189	1102	1508	141	830	1750
1926	365	2	284	1651	2159	217	1262	2556
1927	365	0	248	1417	1812	186	1058	2212
1928	366	0	333	1884	2134	244	1390	2589
1929	365	0	334	1695	2190	245	1242	2567

* Expressed in millionths of the Sun's disc.
† Expressed in millionths of the Sun's hemisphere.

TABLE III.

No. of Rotation.	Rotation commenced G.C.T.	Spots NORTH of the Equator. Mean Daily Area.	Spots NORTH of the Equator. Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean Daily Area.	Spots SOUTH of the Equator. Mean Heliographic Latitude.	Mean Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1007	Dec. 25·85	785	8·65	221	11·46	+4·23	9·27
1008	Jan. 22·19	249	6·55	627	9·49	−4·93	8·66
1009	Feb. 18·53	226	8·28	961	9·57	−6·17	9·33
1010	Mar. 17·86	285	7·73	501	10·97	−4·18	9·79
1011	Apr. 14·14	599	8·02	401	8·72	+1·31	8·30
1012	May 11·39	544	12·07	195	11·38	+5·88	11·89
1013	June 7·60	465	11·60	877	10·46	−2·81	10·85
1014	July 4·80	172	7·76	804	10·55	−7·33	10·06
1015	Aug. 1·01	174	11·78	661	10·24	−5·65	10·56
1016	Aug. 28·24	68	16·21	205	12·14	−5·10	13·15
1017	Sept. 24·50	510	9·04	663	14·40	−4·21	12·07
1018	Oct. 21·79	1210	11·60	658	10·74	+3·73	11·30
1019	Nov. 18·10	2306	12·47	776	8·93	+7·09	11·58

TABLE IV.

Year.	No. of Days on which Sun Photographed.	Spots NORTH of the Equator. Mean Daily Area.	Spots NORTH of the Equator. Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean Daily Area.	Spots SOUTH of the Equator. Mean Heliographic Latitude.	Mean Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1923	365	33	13·88	22	17·32	+ 1·34	15·26
1924	366	232	22·19	45	25·54	+14·49	22·73
1925	365	517	20·70	313	19·37	+ 5·60	20·20
1926	365	663	19·27	599	17·98	+ 1·60	18·66
1927	365	379	17·75	679	13·54	− 2·33	15·05
1928	366	727	11·93	663	15·23	− 1·02	13·50
1929	365	656	10·73	586	10·27	+ 0·82	10·51

Royal Observatory, Greenwich :
1930 *December* 18.

The Problem of n Bodies and the Expansion of the Universe.

By G. C. McVittie, Ph.D.

(*Communicated by Sir A. S. Eddington.*)

I. *Introduction and Summary.*

It has been shown by Lemaître * that the phenomenon known as the recession of the nebulæ can be explained, on the theory of relativity, if the universe is regarded as a three-dimensional sphere of expanding radius. In the infinitely remote past the universe is supposed to have started expanding from the equilibrium state (which Eddington † has shown to be unstable) known as the Einstein Universe. This equilibrium state is characterised by a uniform distribution of matter in the universe whose density is related to the radius of the Einstein universe and the "cosmical constant" λ by well-known formulæ.

But the Lemaître universes are of two types, expanding or contracting, and the work of Lemaître gave no indication as to why the universe, on departing from the equilibrium Einstein state, should begin to expand rather than to contract. In the present paper we show that such an initial expansion could have been started by the formation of condensations (*i.e.* particles of greater density than the surrounding matter) in an Einstein universe.

The problem had already been attacked by Dr. W. H. McCrea and the author,‡ and the result obtained was that if a *single* condensation formed in an Einstein universe, this universe would begin to *contract*. But it was also stated that in the actual universe the case was rather different, since there we have a large number of condensations scattered throughout it at random, and that this might affect the result obtained. This is now found to be the case, in the sense that in treating the problem of n condensations we are led to a different line-element from which the expansion of the universe can be deduced.

The discrepancy between the two results arises essentially from this: that the problem of determining the line-element of a universe which contains both condensations and a uniform distribution of matter, reduces mathematically to finding an appropriate generalization for a curved space of the potential function, $1/r$, of Newtonian theory, and this generalization is to some extent a matter of definition. In the previous paper, for instance, we obtained the generalization by assuming a form of line-element suggested to us by the Schwarzschild solution of Einstein's gravitational equations in an uncurved space. In the present paper, on the other hand, it is arrived at as the elementary solution of the generalization of Laplace's equation to a curved space. These two results are not reducible to each other. Further, it is the integral of this generalized potential function taken over the whole

* G. Lemaître, *Ann. Soc. Scientifique de Bruxelles*, **47** A, 49 (1927 April).
† Sir A. S. Eddington, *M.N.R.A.S.*, **90**, 668 (1930).
‡ W. H. McCrea and G. C. McVittie, *M.N.R.A.S.*, **91**, 128 (1930).

of space which is the determining factor as to whether an expansion or a contraction of the universe sets in. It would seem, therefore, that the conclusion to be drawn from the result previously obtained and the present one is that it is the *type* of condensation which forms that determines the contraction or expansion. The different types of condensations can, of course, be distinguished only by their different potential functions in the same system of co-ordinates.

We mention in § II. three forms of the line-element of a spherical space which are employed in the course of the paper. In § III. the equilibrium solution, to a first approximation, of the problem of n bodies in a universe which also contains a distribution of matter outside the bodies, is given. The result is then employed for the calculation of the volume of a universe of given proper mass in which condensations have formed, starting from an Einstein state. The method of calculation is essentially the same as that given in a previous paper.* The volume is found to be greater than that of the original Einstein state. It is concluded that the formation of condensations of the type considered would cause the Einstein universe to start expanding.

II. *The Line-element of a Lemaître Universe.*

The fundamental form of the line-element of a Lemaître universe we shall take to be

$$ds^2 = dt^2 - a^2(t)\mathrm{R}^2\{d\chi^2 + sin^2\,\chi(d\theta^2 + sin^2\,\theta d\phi^2)\} \quad . \qquad (1)$$

The space dimensions here correspond to a sphere of radius $a(t)\mathrm{R}$ at the instant t. R is a constant, the radius of the Einstein universe from which the Lemaître universe started in the infinitely remote past. The co-ordinate χ measures distance from the origin, and the universe extends from $\chi = 0$ to $\chi = \pi$.

An Einstein universe is one for which $a = 1$, $\dot{a} = 0$, $\ddot{a} = 0$ (dots denote differentiation with respect to t). It is unstable, and the slightest disturbance of the distribution of the matter in it will cause it to become an expanding or contracting universe with the line-element (1). Its line-element is

$$ds^2 = dt^2 - \mathrm{R}^2\{d\chi^2 + sin^2\,\chi(d\theta^2 + sin^2\,\theta d\phi^2)\}.$$

Two other forms of this should be noted. First put

$$r = 2\mathrm{R}\,tan\,\frac{\chi}{2} \quad . \quad . \quad . \quad . \quad . \qquad (1a)$$

and leave θ, ϕ unchanged. We get

$$ds^2 = dt^2 - \frac{\{dr^2 + r^2(d\theta^2 + sin^2\,\theta d\phi^2)\}}{\{1 + r^2/4\mathrm{R}^2\}^2} \quad . \quad . \qquad (2)$$

Secondly, transform from "polar" co-ordinates $(r,\ \theta,\ \phi)$ to

* W. H. McCrea and G. C. McVittie, *loc. cit.*, § IV.

"Cartesians" (x_1, x_2, x_3) by the usual formulæ of Euclidean geometry: we get the symmetrical form

$$ds^2 = dt^2 - \frac{\{dx_1^2 + dx_2^2 + dx_3^2\}}{(1 + r^2/4R^2)^2} \quad . \quad . \quad (3)$$

where

$$r^2 = x_1^2 + x_2^2 + x_3^2.$$

In the next paragraph we determine an equilibrium solution with condensations of the gravitational equations, which departs to the first order only from the form (3).

III. *The Field of n Bodies.*

1. We contemplate a state of affairs in which a finite number n of small condensations have formed in an Einstein universe. These condensations are supposed scattered at random throughout the universe, but, on the average, their distances apart are supposed to be so great that the field of any one particle does not interfere with that of any other. Now it is known * that, under these conditions, the line-element for the field of n bodies in an uncurved space, *i.e.* one in which there is no uniform distribution of matter but only the n bodies, is of the form (in the first approximation)

$$ds^2 = (1 - 2\Omega)dt^2 - (1 + 2\Omega)(dx_1^2 + dx_2^2 + dx_3^2) \quad . \quad (4)$$

where

$$\Omega = \sum_{i=1}^{n} \frac{m_i}{r_i} \qquad \left(\frac{m_i^2}{r_i^2} \text{ neglected}\right). \quad . \quad . \quad (4a)$$

Here m_i is the mass of the ith particle and r_i is its distance from the point considered. Hence we shall assume that in our universe the line-element will be of the form (to a first approximation)

$$ds^2 = (1 - m\alpha)dt^2 - \left(\frac{1}{(1 + r^2/4R^2)^2} + m\beta\right)(dx_1^2 + dx_2^2 + dx_3^2) \quad (5)$$

where α, β will be functions analogous to the potential function $(4a)$ of Newtonian theory, the particles being all of equal mass m.

To simplify the mathematics it is necessary to make the following assumptions:—

(a) m is of the first order and $\dot{m}^2$, etc., are negligible.

(b) $\dot{m}$ (the differential of m with respect to the time) is of the second order and therefore negligible. This means that the rate of formation of a condensation is slow.

(c) The velocities $(\dot{x}_1, \dot{x}_2, \dot{x}_3)$ of a condensation are of the second order, so that, to our order of approximation, the condensations are at rest with respect to the co-ordinates (x_1, x_2, x_3).

(d) α, β do not involve the time explicitly. Hence, in virtue of (c), the time-derivatives of α, β, r^2 are negligible.

* Eddington, *Mathematical Theory of Relativity*, 2nd ed., § 46, p. 101.

With these restrictions, we calculate the non-zero components of the Riemann Christoffel tensor, $G_{\mu\nu}$, for the metric (5). We shall denote the expression $(1 + r^2/4R^2)^{-2}$ by u and the first and second derivatives of α with respect to, say, x_1 by α_1 and α_{11} respectively. Similarly with regard to β and u. Further, we shall employ the operators

$$\nabla^2 \equiv \frac{\delta^2}{\delta x_1^2} + \frac{\delta^2}{\delta x_2^2} + \frac{\delta^2}{\delta x_3^2}$$

and

$$D \equiv \frac{u_1}{u}\frac{\delta}{\delta x_1} + \frac{u_2}{u}\frac{\delta}{\delta x_2} + \frac{u_3}{u}\frac{\delta}{\delta x_3}.$$

We obtain

$$G_{44} = \frac{m}{2u}(\nabla^2 + \tfrac{1}{2}D)\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$G_{ss} = -\frac{2u}{R^2} + \frac{m}{2}\left\{\begin{array}{l}\nabla^2\frac{\beta}{u} + D\left(\frac{\beta}{u} - \frac{1}{2}\alpha\right) \\ + \left(\frac{\beta}{u}\right)_{ss} - \frac{u_s}{u}\left(\frac{\beta}{u}\right)_s + \frac{u_s}{u}\alpha_s - \alpha_{ss}\end{array}\right\} \quad . \quad (7)$$
$$(s = 1, 2, 3)$$

In (7), of course, there is no summation over s.

We now calculate the energy tensor

$$-8\pi T_{\mu\nu} = G_{\mu\nu} - \tfrac{1}{2}(G - 2\lambda)g_{\mu\nu}.$$

We get, for the non-zero components,

$$-8\pi T_4^{\,4} = \lambda - \frac{3}{R^2} + \frac{m}{4}\left\{\frac{4}{u}(\nabla^2 + \tfrac{1}{2}D)\left(\frac{\beta}{u}\right) + \frac{12}{R^2}\frac{\beta}{u}\right\} \quad . \quad (8)$$

$$-8\pi T_s^{\,s} = \lambda - \frac{m}{4u}(\nabla^2 + \tfrac{1}{2}D)\alpha$$
$$+ \frac{1}{u}\left(1 - \frac{m\beta}{u}\right)\left\{\begin{array}{l}-\frac{u}{R^2} + \frac{m}{4}\left(2\nabla^2\left(\frac{\beta}{u}\right) + (\tfrac{1}{2}D - \nabla^2)\alpha - 2\left(\frac{\beta}{u}\right)_{ss}\right. \\ \left. + 2\frac{u_s}{u}\left(\frac{\beta}{u}\right)_s - 2\frac{u_s}{u}\alpha_s + 2\alpha_{ss}\right)\end{array}\right\} \quad . \quad (9)$$
$$(s = 1, 2, 3)$$

Since the components $T_1^{\,4}$, $T_2^{\,4}$, $T_3^{\,4}$ are zero, there is no mass-motion of the matter in the universe. Hence the pressure components ($T_1^{\,1}$, $T_2^{\,2}$, $T_3^{\,3}$) are due to random motions of the particles constituting the matter. Further, since the condensations are scattered at random throughout the universe, the pressure at any point may be taken as being isotropic. This means that

$$T_1^{\,1} = T_2^{\,2} = T_3^{\,3} \quad . \quad . \quad . \quad . \quad (10)$$

From (9), we can satisfy (10) if

$$-\left(\frac{\beta}{u}\right)_{ss} + \frac{u_s}{u}\left(\frac{\beta}{u}\right)_s - \frac{u_s}{u}\alpha_s + \alpha_{ss} = 0 \quad . \quad . \quad (11)$$

We have three equations here, since $s = 1, 2$, or 3. We see that these equations are all satisfied if

$$a = \frac{\beta}{u} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Substituting for β from (12) into (8) and (9), we get that the energy tensor now has the form

$$-8\pi T_4^4 = \lambda - \frac{3}{R^2} + \frac{m}{4}\left\{\frac{12a}{R^2} + \frac{4}{u}(\nabla^2 + \tfrac{1}{2}D)a\right\} \quad . \quad (13)$$

$$-8\pi T_s^s = \lambda - \frac{1}{R^2} + \frac{ma}{R^2} \qquad (s = 1, 2, 3) \quad . \quad . \quad (14)$$

2. The value of a remains arbitrary so far. We can obtain an equation for it by means of the following considerations: In the first place, the operator $(\nabla^2 + \frac{1}{2}D)$ is just the second order differential parameter * for the metric (5) when $m = 0$, *i.e.* it is the parameter for the metric of the undisturbed Einstein universe. It therefore stands in the same relation to the metric (5) as the operator ∇^2 (the parameter of a Euclidean space) does to the metric (4) which represents the field of n bodies in an otherwise empty world. In the second place, our solution (5) must be reducible to the solution (4) when we take $\lambda = 0$ and allow R to become infinite. In this limiting case our value of a must reduce to Ω, and Ω from (4*a*) is a sum of elementary solutions (in the sense of Hadamard †) of Laplace's equation $\nabla^2\Omega = 0$. Hence *we assume that the required value of* a *is a sum of elementary solutions of the generalization of Laplace's equation*, i.e. *of the equation.*

$$(\nabla^2 + \tfrac{1}{2}D)a = 0 \quad . \quad . \quad . \quad . \quad (15)$$

We remark that the name "elementary solution" is given by Hadamard to that solution of a partial differential equation of the second order which

(i) is continuous and differentiable as often as we please in any finite region of space, except in the neighbourhood of a singular point, say (a, b, c);

(ii) becomes infinite to as low an order as possible, at the singular point and on all the isotropic lines through the point.

These are the properties that distinguish the solution

$$\Omega = \{(x-a)^2 + (y-b)^2 + (z-c)^2\}^{-\frac{1}{2}}$$

of Laplace's equation from all the others with a singularity at the point (a, b, c).

By a change of co-ordinates we can reduce the equation (15) to one

* See, *e.g.*, L. P. Eisenhart, *Riemannian Geometry* (Princeton Univ. Press), chap. ii., § 14.

† Hadamard, *Lectures on Cauchy's Problem in Linear Partial Differential Equations* (Yale Univ. Press, 1923), pp. 75 *et seq.*

whose elementary solution is known. For (15) written in full (substituting for u its value in terms of r) is

$$\frac{\delta^2\alpha}{\delta x_1^2}+\frac{\delta^2\alpha}{\delta x_2^2}+\frac{\delta^2\alpha}{\delta x_3^2}-\frac{2}{4R^2+r^2}\left(x_1\frac{\delta\alpha}{\delta x_1}+x_2\frac{\delta\alpha}{\delta x_2}+x_3\frac{\delta\alpha}{\delta x_3}\right)=0.$$

Introducing polar co-ordinates, so that the metric takes the form (2), we reduce this equation to the form

$$\frac{\delta^2\alpha}{\delta r^2}+\frac{\delta R^2 r}{4R^2+r^2}\frac{\delta\alpha}{\delta r}+\frac{1}{\sin\theta}\frac{\delta}{\delta\theta}\left(\sin\theta\frac{\delta\alpha}{\delta\theta}\right)+\frac{1}{\sin^2\theta}\frac{\delta^2\alpha}{\delta\phi^2}=0 \quad . \quad (16)$$

The elementary solution of this equation is known.* If the singularity occurs at the point $r = a_1$, $\theta = b_1$, $\phi = c_1$, this solution is of the form

$$\alpha = \frac{1}{a_1'}\frac{z_1'}{z_1}-\frac{1}{a_1}\frac{z_1}{z_1'} \quad . \quad . \quad . \quad . \quad (17)$$

where

$$\left.\begin{aligned} z_1 &= r^2+a_1^2-2a_1 r\,(\cos\theta\cos b_1+\sin\theta\sin b_1\cos(\phi-c_1))\\ z_1' &= r^2+a_1'^2+2a_1' r\,(\cos\theta\cos b_1+\sin\theta\sin b_1\cos(\phi-c_1)) \end{aligned}\right\} \quad (18)$$

and

$$a_1 a_1' = 4R^2.$$

As in the Newtonian case, we take the condensation to be located at the singular point of the elementary solution; then (17) represents its potential. Hence if we have n condensations whose masses we take, for simplicity, to be all equal, the corresponding value of α is, with an obvious notation,

$$\alpha = \sum_{r=1}^{n}\left(\frac{1}{a_r'}\frac{z_r'}{z_r}-\frac{1}{a_r}\frac{z_r}{z_r'}\right) \quad . \quad . \quad . \quad (19)$$

so that the rth condensation is located at (a_r, b_r, c_r).

We can now write down (in the first approximation) the metric for the field of n bodies (condensations) in a universe where there is also a continuous distribution of matter. In (5) we substitute "polar" co-ordinates for the "Cartesians" (x_1, x_2, x_3), then from (12) we obtain β in terms of α. Finally (19) gives us the value of α. We get

$$ds^2 = \left\{1-m\sum_{r=1}^{n}\left(\frac{1}{a_r'}\frac{z_r'}{z_r}-\frac{1}{a_r}\frac{z_r}{z_r'}\right)\right\}dt^2 - \frac{\left\{1+m\sum_{r=1}^{n}\left(\frac{1}{a_r'}\frac{z_r'}{z_r}-\frac{1}{a_r}\frac{z_r}{z_r'}\right)\right\}}{(1+r^2/4R^2)^2}(dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2) \quad (20)$$

where z_r, z_r', a_r' are given by expressions of the form (18) and m is the mass of any one of the condensations.

* See J. M. Whittaker, *Proc. Camb. Phil. Soc.*, **24**, 414 (1928); and E. T. Copson, *Proc. Roy. Soc.*, A, **118**, 184 (1928).

IV. *The Proper-Mass of the Universe.*

We shall now calculate the proper-mass of the universe whose line-element is (20), and we shall show that, for a given proper-mass, the universe (20) has a greater volume than the Einstein universe with the same proper-mass.

We have, from (13) and (14), the following values for the density of matter (ρ) and the pressure (p) in the universe :—

$$8\pi\rho = \frac{3}{R^2} - \frac{3am}{R^2} - \lambda \quad . \quad . \quad . \quad (21)$$

$$8\pi p = \lambda - \frac{1}{R^2} + \frac{ma}{R^2} \quad . \quad . \quad . \quad . \quad (22)$$

where, in (21), use has been made of (15), so that (21) holds everywhere except at the points where the condensations occur.

From (21) and (22) we get, for the proper density,

$$8\pi\rho^{\times} = 8\pi(\rho - 3p) = -4\lambda + \frac{6}{R^2} - \frac{6ma}{R^2} \quad . \quad . \quad (23)$$

Now the new universe (20) departs from an Einstein universe of radius R only in terms of order m. Hence we can say that the value of λ in terms of R departs from the Einstein value by an amount of the same order. We therefore put

$$\lambda = \frac{1 + \eta}{R^2} \quad . \quad . \quad . \quad . \quad (24)$$

where η is a constant of the order of m.

Using (24), (22) and (23) become

$$8\pi p = \frac{\eta + ma}{R^2} \quad . \quad . \quad . \quad . \quad (22')$$

$$8\pi\rho^{\times} = \frac{2 - 4\eta - 6ma}{R^2} \quad . \quad . \quad . \quad (23')$$

Again, we have already shown that the pressure is due to random motions of the particles of the matter. Let v be the mean random velocity, then

$$p = \tfrac{1}{3}\rho v^2 \quad . \quad . \quad . \quad . \quad (25)$$

The volume-element of the space (20) is

$$dV = r^2 \sin\theta(1 + ma)^{3/2} d\theta d\phi dr/(1 + r^2/4R^2)^3 \quad . \quad (26)$$

where a is given by (19).

But the matter has a mean random velocity v. Hence the volume-element corresponding to the proper-density $\rho^{\times}$ is $dV/\sqrt{1 - v^2}$. Hence, to our order of approximation, the element of proper-mass is

$$\rho^{\times}(1 + \tfrac{1}{2}v^2)dV = (\rho^{\times} + \tfrac{3}{2}p)dV \quad . \quad . \quad . \quad . \quad (27)$$

since, in using (25), we need not distinguish between ρ and $\rho^{\times}$.

Substituting from (26), (22′), and (23′) into (27) and neglecting powers of m higher than the first, we get for the element of proper-mass,

$$\frac{1}{4\pi}(1 - \tfrac{5}{4}\eta - \tfrac{3}{4}m\alpha)r^2 \sin\theta d\theta d\phi dr/\mathrm{R}^2(1 + r^2/4\mathrm{R}^2)^3 \quad . \quad (28)$$

The value of α from (19) involves n singular points. If we exclude these points by taking small spheres around them, we can obtain from (28) the proper-mass of the universe *excluding* the masses of the condensations themselves. This mass is M, where

$$\mathrm{M} = \frac{1}{4\pi}(1 - \tfrac{5}{4}\eta)\int r^2 \sin\theta d\theta d\phi dr/\mathrm{R}^2(1 + r^2/4\mathrm{R}^2)^3 - \frac{3}{16\pi}m\oiint \alpha r^2 \sin\theta d\theta d\phi dr/\mathrm{R}^2(1 + r^2/4\mathrm{R}^2)^3 \quad . \quad (29)$$

where the integral signs denote triple integrals.

In the first integral the integration is over the whole universe, whilst in the second it is over those parts which are not excluded by the spheres around the singular points.

The calculation of the first integral is immediate: it is the same as that giving the mass of an Einstein universe of radius R. With regard to the second integral we see, from (19), that it is really a sum of integrals, each integral giving the "perturbation" of the total mass due to one of the condensations. Further, it is seen that to evaluate these integrals would not be easy, with the exception of the integral corresponding to a condensation situated at the origin of co-ordinates. But since all the condensations are supposed equal and the values of the integrals depend on the *whole* universe and cannot therefore be affected by the choice of the origin of co-ordinates, we may say that the value of the second integral in (29) is equal to n times the value of the integral arising from the condensation at the origin alone.

From (19) and (18) the term in α corresponding to the condensation at the origin is

$$\frac{1}{r} - \frac{r}{4\mathrm{R}^2} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Hence the integral is

$$\int_0 \int_0^{\pi} \int_0^{2\pi} \frac{r^2\left(\frac{1}{r} - \frac{r}{4\mathrm{R}^2}\right)}{\mathrm{R}^2(1 + r^2/4\mathrm{R}^2)^3} dr \sin\theta d\theta \,.\, d\phi \quad . \quad . \quad (31)$$

The limits of r are obtained from (1a) and correspond to the range $0 \leqslant \chi \leqslant \pi$. The lower limit of r may be taken to be zero since we may suppose that the radius of the sphere surrounding the condensation at the origin is of order m, so that the lower limit in (31) would give a zero contribution when (31) is introduced into (29).

Evaluating the integral (31) we see that its value is zero.

Hence we have the result that

$$\mathrm{M} = (1 - \tfrac{5}{4}\eta)\frac{\pi}{2}\mathrm{R}.$$

Adding to this the masses of the condensations themselves we get for the total proper-mass of the universe,

$$M^{\times} = (1 - \tfrac{5}{4}\eta)\frac{\pi}{2}R + nm \quad . \quad . \quad . \quad (32)$$

V. *The Cosmical Constant, λ.*

We have supposed that originally the universe was an Einstein one. Hence the Einstein relation must hold between the proper-mass of the universe, which is supposed conserved during the formation of the condensations, and the absolute cosmical constant λ. This gives

$$M^{\times} = \frac{\pi}{2}\frac{1}{\sqrt{\lambda}} = \frac{\pi}{2}R(1 - \tfrac{1}{2}\eta) \quad . \quad . \quad . \quad (33)$$

by (24)
Hence (32) and (33) give

$$(1 - \tfrac{5}{4}\eta)\frac{\pi}{2}R + nm = \frac{\pi}{2}R(1 - \tfrac{1}{2}\eta)$$

or

$$\eta = \frac{8nm}{3\pi R}.$$

Hence from (32)

$$M^{\times} = \frac{\pi}{2}R - \tfrac{2}{3}nm \quad . \quad . \quad . \quad . \quad (34)$$

VI. *The Expansion of the Universe.*

The total volume of the universe is obtained by evaluating (26) taken over the whole universe. Hence

$$V = \int \frac{r^2 \sin\theta d\theta d\phi dr}{(1 + r^2/4R^2)^3} + \tfrac{3}{2}m\oint \frac{\alpha r^2 \sin\theta d\theta d\phi dr}{(1 + r^2/4R^2)^3},$$

where the integrals have the same meanings as in the case of (29). Evaluating them as in that case we obtain

$$V = 2\pi^2 R^3.$$

Substituting for R in terms of the proper-mass from (34), we get

$$V = \frac{16}{\pi}M^{\times 3}\left(1 + \frac{2nm}{M^{\times}}\right),$$

which is greater than the volume $(16M^{\times 3}/\pi)$ of the original Einstein universe. Hence the volume of the equilibrium state, with condensations, of the universe is greater than the volume of the original Einstein state.

The conclusion we arrive at is that if condensations of the type we have considered begin to form in an Einstein universe, it must increase in volume. But any departure from equilibrium will cause an Einstein

universe to become a Lemaître one with line-element (1), and in such a universe an initial expansion guarantees that the universe continues to expand indefinitely. Hence the formation of these condensations will provide a mechanism for starting the expansion of the universe which is required to account for the recession of the nebulæ.

The Transmutation of Elements in Stars. By A. H. Wilson, M.A.

(*Communicated by R. H. Fowler, F.R.S.*)

1. *Introduction.*

The source of stellar energy is one of the outstanding problems of astrophysics and seems to be as far as ever from solution. There are three main processes to be considered, which are, in order of importance, the annihilation of electrons and protons, the synthesis of helium, and the building up of complex elements. Until recently it was not possible to obtain any well-founded ideas as to the frequency of occurrence of these processes, but modern quantum mechanics has enabled a start to be made on two of the processes. The rate of annihilation of electrons and protons has been calculated on the most recent theory,* and has been shown to be inconsistent with the known stability of matter. At the moment, therefore, it is impossible to draw any conclusions of astrophysical interest from this theoretical development. The synthesis of helium is wrapped in obscurity and seems to be of an entirely different nature from the synthesis of the heavier elements. The last and, from an astrophysical point of view, least important process is the only one which can be safely treated by the quantum theory in its present state. It is, however, of interest to obtain some idea of the conditions under which the building up of elements can occur, as the annihilation of matter can hardly be expected to take place when even the transmutation of elements is impossible. It is the object of the present paper to examine these conditions.

The first quantitative results on the building up of atoms in stars were obtained by Atkinson and Houtermans,† and were based on some speculations by Gamow on artificial disintegration which had no sure foundation. If the calculations are carried out properly, it is found that the value given by these authors for the rate of transformation is incorrect, though of the right order for light nuclei.

There are essentially three processes by which atoms can be built up—the radiative capture of a proton by a nucleus, the radiative capture of an α-particle, and the capture of an α-particle together with the simultaneous emission of a proton. The first process is more rapid than the second on account of the smaller mass and charge of the proton compared with the α-particle, while the rate of the third process is limited by the rate of production of nuclei containing protons. It will therefore be sufficient to consider the radiative capture of a proton by a nucleus.

* P. A. M. Dirac, *Proc. Camb. Phil. Soc.*, **26**, 361 (1930).
† *Z. f. Physik*, **54**, 656 (1929).

We consider a stream of protons incident on a nucleus, which is taken to be of a simplified form, and calculate the cross-section for radiative capture into a stable orbit inside the nucleus. This is a much more logical procedure than that adopted by Atkinson and Houtermans, who base their arguments on the penetrability of the potential barrier of the nucleus. This gives rise to difficulties when protons are considered which have energies near a resonance energy of the nucleus.*

On inserting this value of the cross-section into the usual collision formulæ, we obtain the "rate of reaction." It is found that, if the proton has not sufficient energy to pass over the top of the potential barrier of the nucleus, the cross-section is very small. If, however, the nucleus possesses a resonance level,* then the cross-section is enormously increased for protons with energies near the resonance energy. This is counterbalanced by the small width of the resonance level. If the proton has sufficient energy to pass right over the top of the potential barrier, the cross-section is of the same order as the classical cross-section of the nucleus.

The rate of reaction depends mainly on an exponential factor, which for normal captures depends on the penetrability of the potential barrier. It is for this reason that Atkinson and Houtermans obtain the correct order of magnitude for light nuclei. The rate of reaction is very sensitive to the temperature of the star, and at the temperature 4×10^7 degrees given by Eddington in his *Internal Constitution of the Stars*, the reaction rate is negligible. At the temperature 10^{11} given by Milne's new theory,† the protons have sufficient energy to pass over the potential barrier and so the rate of reaction is large.

2. *Properties of a Simplified Nucleus.*

2.0. Most of the results in this section have already been obtained by Fowler (R. H.) and Wilson, and are merely summarised here with a few necessary additions.

We consider a spherically symmetrical nucleus, the potential energy U of a proton in the presence of the nucleus being that shown in the figure.

The wave function defining the state of the proton satisfies the equation

$$\nabla^2\psi + \kappa^2(\mathrm{E} - \mathrm{U})\psi = 0 \qquad \left(\kappa^2 = \frac{8\pi^2\mu}{h^2}\right) \quad . \quad . \quad (1)$$

where μ is the mass of the proton. Assuming a solution of the form

$$\psi = \frac{1}{r}\chi_l(r)\mathrm{P}_l(\cos\theta), \quad . \quad . \quad . \quad (2)$$

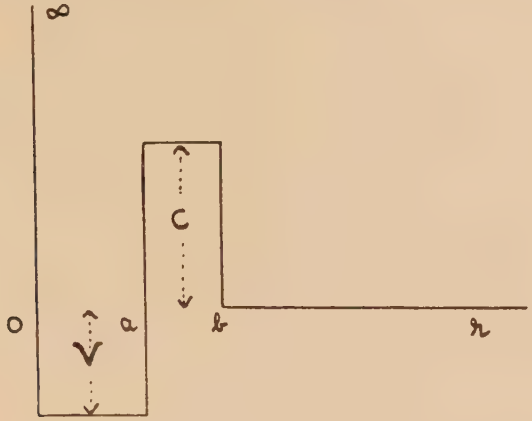
we have

$$\frac{d^2\chi}{dr^2} + \left\{\kappa^2(\mathrm{E} - \mathrm{U}) - \frac{l(l+1)}{r^2}\right\}\chi = 0 \quad . \quad . \quad (3)$$

* Fowler and Wilson, *Proc. Roy. Soc.*, A, **124**, 493 (1929).
† *Nature*, **126**, 238 (1930).

If V is sufficiently large, this equation has both a discrete and a continuous set of solutions, which we discuss separately.

2.1. If E >o we have the continuous set of solutions. The equation can be solved exactly in terms of Bessel functions, but we shall confine



ourselves to the discussion of the equation with $l = 0$,* as only this case is used in the subsequent work. The solution then represents a spherical stream of protons incident on the nucleus and being reflected.

We have

$$\chi = A \sin \kappa\sqrt{(E + V)}r \qquad (r < a).$$
$$\chi = Be^{-\kappa\sqrt{(C-E)}(r-a)} + B_1 e^{\kappa\sqrt{(C-E)}(r-a)} \qquad (a < r < b).$$
$$\chi = De^{-i\kappa\sqrt{E}(r-b)} + D_1 e^{i\kappa\sqrt{E}(r-b)} \qquad (b < r).$$

The equations of continuity at $r = a$ and $r = b$ are

$$A \sin \kappa\sqrt{(E + V)}a = B + B_1$$
$$A \cos \kappa\sqrt{(E + V)}a = (-B + B_1)\sqrt{\frac{C - E}{E + V}}$$
$$D + D_1 = B/\vartheta + B_1\vartheta$$
$$i(D - D_1)\sqrt{\frac{E}{C - E}} = B/\vartheta - B_1\vartheta,$$

where
$$\vartheta = \exp\{\kappa\sqrt{(C - E)}(b - a)\} \quad . \quad . \quad . \quad (4)$$

Solving these equations we find that D and D_1 are conjugate complex quantities, and that

$$D = \tfrac{1}{4}A\left[\vartheta\left\{1 + i\sqrt{\frac{C - E}{E}}\right\}\left\{\sin \kappa\sqrt{(E + V)}a + \sqrt{\frac{E + V}{C - E}}\cos \kappa\sqrt{(E + V)}a\right\}\right.$$
$$\left. + \frac{1}{\vartheta}\left\{1 - i\sqrt{\frac{C - E}{E}}\right\}\left\{\sin \kappa\sqrt{(E + V)}a - \sqrt{\frac{E + V}{C - E}}\cos \kappa\sqrt{(E + V)}a\right\}\right] \quad (5)$$

* We are therefore considering only direct hits, in which l, the quantised angular momentum of the proton about the centre of the nucleus, is zero.

In general $|\,A/D\,|^2$ is proportional to ϑ^{-2}, and without serious error we may put

$$\left|\frac{A}{D}\right|^2 = 16\frac{E}{C}\vartheta^{-2} \quad . \quad . \quad . \quad . \quad (6)$$

If, however, E happens to be a solution of

$$\tan \kappa\sqrt{(E_m + V)}a = -\sqrt{\frac{E_m + V}{C - E_m}} \quad . \quad . \quad (7)$$

then $|\,A/D\,|^2$ is of order ϑ^2, the precise value being

$$\left|\frac{A}{D}\right|^2 = 4\frac{E_m(C + V)}{C(E_m + V)}\vartheta^2 \quad . \quad . \quad . \quad (8)$$

Such an energy level is called a resonance level, and is not absolutely sharp, the width being of the order E_m/ϑ^2.

2.2. If $E < 0$ the proton is confined to the interior of the nucleus, and the wave function is effectively different from zero only when $r < a$. There is now no possibility of $C - E$ becoming small, and the results will not be affected much by taking C to be infinite. We then have a solution

$$\psi = \frac{1}{r}\chi_l(r)P_l(\cos\theta),$$

where

$$\chi_l = A_n\{\kappa r\sqrt{(E_n + V)}\}^{\frac{1}{2}}J_{l+\frac{1}{2}}\{\kappa r\sqrt{(E_n + V)}\} \qquad (r < a) \quad . \quad (9)$$

and $\chi_l = 0$ for $r > a$. The energy values are given by the negative roots of

$$\kappa a\sqrt{(E_n + V)} = \alpha_{n-l},$$

where α_{n-l} is the $(n - l)$th positive root of $J_{l+\frac{1}{2}}(x) = 0$.

The lowest state is given by $l = 0$ and $n = 1$, which gives

$$E_1 = -V + \pi^2/\kappa^2a^2.$$

The next lowest state is given by $l = 1$ and $n = 2$, the wave function being

$$\psi_2 = \frac{A_2}{r}\left\{\kappa r\sqrt{(E_2 + V)}\right\}^{\frac{1}{2}}J_{3/2}\{\kappa r\sqrt{(E_2 + V)}\}P_1(\cos\theta) \qquad (10)$$

If we normalise the wave functions to unity we have

$$A_1^2 = (2\pi a)^{-1}$$

$$A_2^2 = \frac{3}{2\pi a^2\kappa\sqrt{(E_2 + V)}J'^2_{3/2}\{\kappa a\sqrt{(E_2 + V)}\}} \quad . \quad . \quad (11)$$

2.3. We have yet to normalise the wave functions of positive energy, which we do in the following way. If we have a stream of protons in the direction of the x-axis, the normalising factor must be chosen so that at large distances from the nucleus the incident stream represents one proton crossing unit area in unit time. This gives

$$\psi = \left(\frac{\mu}{2E}\right)^{\frac{1}{4}}e^{i\kappa\sqrt{E}x} \quad . \quad . \quad . \quad . \quad (12)$$

The well-known expansion in terms of Legendre functions gives

$$e^{i\kappa\sqrt{\mathrm{E}}x} = e^{i\kappa\sqrt{\mathrm{E}}r\cos\theta} = \sqrt{\frac{2\pi}{\kappa\sqrt{\mathrm{E}}r}}\sum_{l=0}^{\infty}(i)^l\frac{2l+1}{2}J_{l+\frac{1}{2}}(\kappa\sqrt{\mathrm{E}}r)\mathrm{P}_l(\cos\theta), \quad (13)$$

the first term being

$$\frac{2}{\kappa\sqrt{\mathrm{E}}r}\sin(\kappa\sqrt{\mathrm{E}}r).$$

The proper normalisation for the wave functions of section 2.1 is therefore

$$|\mathrm{D}|^2 = \frac{1}{\kappa^2\mathrm{E}}\sqrt{\frac{\mu}{2\mathrm{E}}}. \quad . \quad . \quad . \quad . \quad (14)$$

In general

$$|\mathrm{A}|^2 = \frac{h^2}{\pi^2\mathrm{C}}\sqrt{\frac{2}{\mu\mathrm{E}}}\vartheta^{-2}, \quad . \quad . \quad . \quad (15)$$

but for a resonance level

$$|\mathrm{A}|^2 = \frac{h^2(\mathrm{C}+\mathrm{V})}{4\pi^2\mathrm{C}(\mathrm{E}_m+\mathrm{V})}\sqrt{\frac{2}{\mu\mathrm{E}_m}}\vartheta^2. \quad . \quad . \quad (16)$$

3. *The Cross-Section for Capture.*

3.0. If a proton strikes a nucleus it may be captured and emit a γ-ray. As this is a radiative transition the azimuthal quantum number l must change by one unit. We ought, therefore, to split up (12) into its harmonic components, which are given by (13), calculate the probability of transition to a bound state, and sum for all values of l. The simplifications we have already made do not justify us in carrying the work to this degree of accuracy, and it will be sufficient for our purposes to consider a transition from a state of positive energy with $l = 0$ to the discrete state E_2 with $l = 1$. The important factor is ϑ^2, and is the only one which needs to be estimated with some accuracy.

3.1. The matrix component of the polarisation for the radiative transition is $\mathrm{I} = e\int x\psi(\mathrm{E})\psi_2 d\tau$, where e is the charge on the proton and the integration is over the whole range of the co-ordinates.

$$\mathrm{I} = 2\pi e\int_0^{\pi}\sin\theta\cos^2\theta d\theta\int_0^{a} r\mathrm{A}\sin\kappa r\sqrt{(\mathrm{E}+\mathrm{V})}\,.$$
$$\mathrm{A}_2\{\kappa r\sqrt{(\mathrm{E}_2+\mathrm{V})}\}^{\frac{1}{2}}\mathrm{J}_{3/2}\{\kappa r\sqrt{(\mathrm{E}_2+\mathrm{V})}\}dr. \quad (17)$$

This can be evaluated exactly, but it will be sufficient to obtain an upper bound, which is also the order of magnitude, by replacing $\{\kappa r\sqrt{(\mathrm{E}_2+\mathrm{V})}\}^{\frac{1}{2}}\mathrm{J}_{3/2}\{\kappa r\sqrt{(\mathrm{E}_2+\mathrm{V})}\}$ by $\sqrt{(2/\pi)}$, and $\sin\kappa r\sqrt{(\mathrm{E}+\mathrm{V})}$ by 1. The error introduced by these approximations is about the same as that due to taking A^2 as given by (6), and is in the opposite direction. We also use $4/a$ as an approximate value of $\mathrm{A}_2{}^2$.

With these approximations we have

$$I = \tfrac{2}{3}\pi e a^2 A A_2. \quad . \quad . \quad . \quad . \quad (18)$$

Now the radiation emitted per second in the transition is

$$\frac{4}{3}\frac{(2\pi\nu)^4}{c^3} I^2,$$

where $h\nu = E - E_2$. Therefore the number of quanta of γ-rays emitted per second is

$$\frac{4}{3}\frac{(2\pi\nu)^4}{h\nu c^3} I^2. \quad . \quad . \quad . \quad . \quad (19)$$

This is also the number of protons absorbed per second. Owing to the normalisation of $\psi(E)$, (19) represents an area, which is the area a proton must hit in order to be absorbed. If σ is the "diameter for capture," then in general

$$\sigma^2 = \frac{2^{12}\sqrt{2}\pi^3 a^3 e^2}{27 c^3 h^2 \sqrt{\mu}} \frac{(E - E_2)^3}{C\sqrt{E}} \vartheta^{-2}. \quad . \quad . \quad (20)$$

For a proton whose energy lies within a range $E_m(1 - \frac{1}{2}\vartheta^{-2})$, $E_m(1 + \frac{1}{2}\vartheta^{-2})$, where E_m is a resonance energy,

$$\sigma^2 = \frac{2^{10}\sqrt{2}\pi^3 a^3 e^2}{27 c^3 h^2 \sqrt{\mu}} \frac{(E_m - E_2)^3 (C + V)}{C\sqrt{E_m(E_m + V)}} \vartheta^2. \quad . \quad . \quad (21)$$

3.2. We can only make guesses at the quantities E_2, E_m, C, a and b, but inaccuracies in these quantities will not matter much except for the exponential term ϑ. In this term small differences in the assumed values will greatly alter the result, and it is necessary to adopt a more exact model of a nucleus than the one we have been so far using.

In an actual atomic nucleus of atomic number Z the potential is Ze/r for values of $r > 10^{-12}$ cm., and for smaller values it diminishes in some unknown fashion. We shall suppose that the nuclear potential is Ze/r for $r > d$, and $-V/e$ for $r < d$, where $d = o(10^{-13})$. Then, as is well known,* the exponential factor ϑ has to be replaced by

$$\exp \kappa \int_d^{r_0} \sqrt{\left(\frac{Ze^2}{r} - E\right)} dr, \quad . \quad . \quad . \quad (22)$$

where r_0 is the zero of $E - Ze^2/r$.

If E is not too large the integral is independent of d, and (22) becomes

$$\exp \frac{\pi\kappa Ze^2}{2\sqrt{E}} \quad . \quad . \quad . \quad . \quad (23)$$

the condition for its validity being $E << Ze^2/d$.

Pose's experiments on the artificial disintegration of aluminium † show that E_2 and E_m are both of the order of a few million volts, and we

* See, for example, R. d'E. Atkinson and F. G. Houtermans, *loc. cit.*

† *Z. f. Physik*, **64**, 1 (1930).

therefore assume as reasonably small values, $-E_2 = 10^{-6}$ ergs, $C = Z10^{-6}$ ergs, $a = 10^{-13}$ cm. We then obtain from (20)

$$\sigma^2 = \frac{10^{-32}}{Z\sqrt{E}} \exp\left(-\frac{Z10^{-3}}{\sqrt{E}}\right). \quad . \quad . \quad . \quad (24)$$

provided $$E << Z10^{-6} \text{ ergs}. \quad . \quad . \quad . \quad . \quad (25)$$

4. *Application to Stellar Interiors.*

4.0. The number of effective collisions between protons and nuclei of atomic number Z per unit volume per unit time in which the kinetic energy of the relative motion lies between E and E + dE is

$$\frac{2N_1N_2}{(kT)^{3/2}}\sigma^2\left(\frac{2\pi}{\mu}\right)^{\frac{1}{2}} e^{-E/kT}E dE, \quad . \quad . \quad . \quad (26)$$

where N_1, N_2 are the molecular densities of protons and nuclei respectively, μ is now the reduced mass of the proton, and T is the temperature of the interior of the star. σ^2 has three different forms according as (1) E is greater than C, (2) E is less than C and is not a resonance level, (3) E is less than C and is a resonance level. We shall treat these cases separately.

4.1. The normal process of capture occurs when E is less than C and is not a resonance level. From (24) and (26) the number of captures per unit volume per unit time is

$$\frac{2N_1N_2}{(kT)^{3/2}}\left(\frac{2\pi}{\mu}\right)^{\frac{1}{2}}\frac{10^{-32}}{Z}\int_0^\infty E^{\frac{1}{2}}e^{-\frac{E}{kT}-\frac{Z10^{-3}}{\sqrt{E}}}dE. \quad . \quad . \quad (27)$$

The upper limit ought really to be an energy of the order of C, but the error involved in taking it to be infinity is negligible except for extremely high temperatures. A similar integral was evaluated by Atkinson and Houtermans, and on adopting their method of calculation (27) reduces to

$$N_1N_2\frac{10^{-19}}{Z}(\xi^2 + \tfrac{1}{6})e^{-3\xi^2}, \quad . \quad . \quad . \quad (28)$$

where the numerical values of the atomic constants have been substituted, and where

$$\xi = \left(\frac{Z10^5}{2\sqrt{T}}\right)^{\frac{1}{3}}. \quad . \quad . \quad . \quad . \quad (29)$$

4.2. For protons with energies near a resonance level of the nucleus the diameter is given by (21) and is very large. On the other hand, the width of a resonance level is very small, of the order E_m/ϑ^2. The number of protons captured through a resonance level per unit volume per unit time is therefore found by substituting this value of σ^2 in (26) and by taking ΔE to be E_m/ϑ^2. This gives

$$N_1N_2\frac{10^{-5}}{T^{3/2}}e^{-\frac{\lambda 10^{10}}{T}}, \quad . \quad . \quad . \quad . \quad (30)$$

where λ is a positive constant of order 1. We are here assuming that the resonance levels of all nuclei are of the order of 10^{-6} ergs. This is to be expected since the energies of the resonance levels only depend on the interior of the nucleus and so should show little variation with atomic number. The experiments on artificial disintegration also show little difference in order of magnitude between the resonance energies of aluminium and of the radioactive elements. Our assumption therefore seems justified.

4.3. If the proton has sufficient energy to pass right over the top of the potential barrier of the nucleus, the exponential factor in (24) is replaced by a factor of order unity. The energy of the proton must now be greater than C, so we may substitute $E = C$ in the formula for σ^2 without making a serious error. This gives $\sigma^2 = Z^{-3/2}\ 10^{-29}$ cm. The number of captures per unit volume per unit time is then obtained by using this value of σ^2 in (26) and integrating from C to infinity, the final result being

$$2\frac{10^{-29}}{Z^{3/2}}\left(\frac{2\pi kT}{\mu}\right)^{\frac{1}{2}} e^{-\frac{C}{kT}}\left(\frac{C}{kT}+1\right). \quad . \quad . \quad . \quad (31)$$

4.4. At the temperature given by Eddington, 4×10^7, the chance of capture is negligible except for the very lightest nuclei, the protons only being able to leak through the potential barrier if it is not too high. At this temperature only a negligible fraction of the protons have sufficient energy to surmount the barrier or even to pass through a resonance level. Except at very high temperatures the number of protons captured which have passed over the top of the barrier is negligible compared with the number captured by the other two processes.

When the temperature is low the normal capture process predominates, but, as stated before, the effect is too small to be observed. As the temperature rises the process of capture through resonance levels becomes increasingly important. Taking $Z = 10$, even at a temperature of 10^9 the resonance captures are already a thousand times more frequent than the normal captures. With increasing atomic number the discrepancy becomes still larger. At the still higher temperatures of the order 10^{11} given by Milne's new theory the protons have sufficient energy to pass over the potential barrier, and capture by this process will predominate even over resonance captures owing to the occurrence of the factor $T^{\frac{1}{2}}$ in (31) as against $T^{-3/2}$ in (30). Taking $T = 10^{11}$ and $Z = 100$, the number of captures per unit volume per unit time is of the order $N_1 N_2 10^{28}$. We therefore conclude that at temperatures of the order 10^9 the main process is capture by resonance, while at temperatures of the order 10^{11} the protons which are captured have passed right over the top of the potential barrier.

In view of the fact that a temperature of the order 10^{10} or 10^{11} is necessary for the reaction rate to become reasonably large, it is difficult to escape the conclusion that some modification of the theory of stellar interiors, such as that proposed by Milne, is essential.

The Shortt Clocks at Greenwich in 1930. By J. Jackson, M.A., D.Sc., and W. Bowyer.

(*Communicated by the Astronomer Royal.*)

In previous papers we have discussed the performance of the two sidereal Shortt clocks No. 3 and No. 11 up to the end of 1929. This paper continues the discussion for the year 1930.

In the autumn of 1929, Shortt No. 3 was opened and a new invar bob of 22 lb. weight was substituted for the type-metal bob of 14 lb. weight. It was hoped that this might remove the secular change in the rate of the pendulum, and although for a month or two at the beginning the secular term could not be detected with certainty, experience with the clock for a year has shown that although the secular term has been considerably reduced it is still more than half its original value.

There has been very little change of temperature in the clock-room, the maximum temperature being about 64°·2 F. at the beginning of September, while the control temperature, below which the temperature did not fall by more than one or two tenths of a degree, was 62°·8. Consequently there is little material for the determination of a temperature coefficient for the pendulum with the altered bob. A solution based on the clock error at four dates and involving four arbitrary constants gave the following formula for the clock error :—

$$-2^{s}\cdot308 - 13^{s}\cdot084t + 1\cdot023t^{2} - 0^{s}\cdot480\int(T - 62^{\circ}\cdot8)dt + \text{nutation} \quad . \quad (7)$$

where t is measured in units of 100 days from mean noon on 1930 February 1. The sign of the temperature coefficient indicates over-correction, and is opposite to that previously determined. The integral taken over the whole year is only $0^{s}\cdot32$. The enumeration of the equations is in continuation of that in *M.N.*, **89**, 239, 1929.

When the temperature changes were neglected the following formula was obtained :—

$$-2^{s}\cdot311 - 13^{s}\cdot174t + 1\cdot022t^{2} + \text{nutation} \quad . \quad . \quad (8)$$

It will be seen that the secular term is hardly affected by the introduction or neglect of a temperature term. Although formula (7) gives a better representation of the errors it has been considered better to extrapolate with formula (8).

Shortt No. 11 has now had an undisturbed run of about $4\frac{1}{2}$ years. The comparison of the error is with that computed from formula (6) (*M.N.*, **89**, 243, 1929), based on observations up to the end of 1928, *i.e.* extrapolated two years.

The departure of the clock from prediction in these two years may be summarised as follows :—

(1) 1929 January to August.—The clock went steadily about $0^{s}\cdot011$ per day slow on the predicted rate, so that by the end of this period the clock was about 2^{s} behind the predicted position.

(2) For about six weeks the clock followed the predicted rate very closely.

(3) 1929 October to 1930 December.—The clock has gone steadily about $0^s{\cdot}009$ per day fast on the predicted rate, and by the end of this period the clock was about 2^s ahead of the predicted position.

The total change in the daily rate of Shortt 11 in $4\frac{1}{2}$ years has been $0^s{\cdot}56$ as compared with $0^s{\cdot}57$ computed from formula (6), which corresponds to a change of $^s{\cdot}035$ in the daily rate per 100 days.

The following table gives a comparison of Shortt 3 with formula (8) above and of Shortt 11 with formula (6) of *M.N.*, **89**, 243, 1929. This table constitutes a continuation of Table I. of *M.N.*, **90**, 270, 1930.

Comparison of Shortt No. 3 with Formula (8) *of this Paper and of Shortt No.* 11 *with Formula* (6) *of M.N.*, **89**, 243.

1930.	No. 3 O −C_8.	No. 11 O—C_6.	Temp.† Corr. for No. 11.
	s	s	s
Jan. 11	−·012	+1·187	−·003
17	+·011	1·144	4
25	+·018	1·100	6
30	−·028	1·010	7
Feb. 5 *	−·062	0·914	8
6 *	+·094	1·053	8
9	+·018	0·957	9
14	+·012	·902	9
20	+·077	·888	10
27	+·060	·809	12
Mar. 8	+·029	·700	8
17	+·071	·662	8
21	+·059	·604	9
29	+·100	·566	9
Apr. 2	+·084	·506	9
9	+·111	·473	8
13	+·116	·444	7
18	+·158	·454	7
22	+·150	·423	6
27	+·160	·401	5
30	+·150	·367	5
May 14	+·170	·265	5
18	+·147	+ ·224	− 5

* Feb. 5, 6. Observations half weight.

† The temperature correction is computed by formula (6) as $^s{\cdot}00252$ per day per 1° F. The quantity given here is the integrated effect from the beginning of 1930. The correction for the whole of 1929 was $+^s{\cdot}163$, approximately the same as for 1930. It will be seen that the temperature corrections are small compared with the total residual errors of the formulæ.

Comparison of Shortt Nos. 3 *and* 11 *with Formulæ.*—continued.

1930.	No. 3 O—C_8.	No. 11 O—C_8.	Temp.* Corr. for No. 11.
	s	s	s
May 28	+·107	+ ·134	− 3
June 4	+·099	·072	−·001
11	+·116	·045	+·001
14	+·087	+ ·006	2
19	+·075	− ·028	2
25	+·039	·111	2
29	+·013	·140	2
July 3	−·003	·156	3
7	−·022	·192	7
17	−·003	·224	29
19	+·008	·223	33
Aug. 2	−·073	·394	43
6	−·092	·436	46
14	−·113	·495	56
16	−·092	·506	58
21	−·126	·541	63
27	−·140	·584	67
30	−·147	·601	·072
Sept. 4	−·108	·610	·089
8	−·040	·585	·103
14	−·049	·670	·122
18	+·002	·647	·133
24	−·063	·776	·142
Oct. 2	−·047	·848	·150
9	−·065	·981	·151
12	−·033	1·016	·152
16	−·081	1·110	·153
21	−·048	1·127	·154
26	−·098	1·250	·155
Nov. 4	−·127	1·401	·157
10	−·111	1·494	·158
16	−·111	1·565	·159
23	−·102	1·610	·160
Dec. 8	−·034	1·744	·162
12	−·039	1·799	·163
21	−·035	1·917	·164
27	+·009	1·962	·166
31	+·015	−2·017	+·167

* See note † on p. 292.

New Double Stars. By T. E. Espin and W. Milburn.

The following new double stars have been found and measured with the 24-inch and $17\frac{1}{4}$-inch Calver reflectors during the year 1930. The arrangement of the lists is similar to that of the previous ones.

Espin No.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.		Nts.	Date 1930.	
	°	h m	° ′	°	″					
2401	..	0 21·3	+35 6	357·4	8·01	11·0	11·1	3	+·022	
2402	..	1 2·4	31 21	233·7	3·26	9·8	12·0	3	·035	
2403	+31·284	32·0	31 18	277·3	6·62	9·5	11·0	3	·370	
	+31·289	33·8	31 40	339·0	15·91	9·2	9·2	2	·522	
2404	+30·318	55·1	31 12	324·1	10·38	8·9	14·5	4	·532	
2405	+31·352	55·4	31 18	267·2	7·10	8·5	10·5	2	·057	
2406	..	2 13·5	31 40	205·6	3·70	10·0	12·0	2	·934	
2407	+32·443	22·1	32 24	278·3	5·83	9·6	9·6	2	·528	
2408	+32·472	29·5	32 56	357·3	7·91	9·5	11·0	4	·761	
2409	+32·476	32·4	32 42	72·2	10·14	9·5	14·0	2	·005	(1931)
2410	..	4 39·2	35 4	255±	6±	11·0	11·0	1	·052	
2411	+32·2165	11 30·5	32 7	327·7	5·87	9·5	12·0	1	·328	
2412	+33·2137	33·8	32 58	328·6	2·24	9·7	9·8	1	·328	
2413	+33·2289	12 53·8	33 25	349·8	5·79	9·5	11·0	1	·364	
2414	..	13 28·5	34 33	268·1	2·45	10·5	10·5	1	·364	
2415	+34·2461	47·5	34 29	63·4	5·83	9·5	11·5	1	·402	
2416	+34·2548	14 10·1	34 40	184·8	2·54	9·5	10·0	1	·364	
2417	..	15 8·2	34 42	168·4	3·75	10·0	10·2	1	·402	
2418	..	18 23·0	32 7	230·5	4·71	9·6	10·5	3	·681	
2419	..	30·6	32 0	157·3	3·54	11·0	11·0	3	·628	
2420	+31·3294	30·6	31 55	175·0	5·06	9·5	9·5	3	·628	
2421	+30·3340	50·3	30 53	66·5	6·63	9·3	10·0	2	·782	
2422	+31·3407	53·0	31 4	174·8	5·31	9·0	12·0	2	·698	
2423	..	19 9·4	31 15	195·1	3·06	10·5	10·5	2	·652	
2424	..	32·9	30 40	348·0	4·79	10·0	10·2	3	·818	
2425	..	35·4	30 52	29·3	4·41	10·0	11·0	3	·775	
2426	+31·3769	42·8	31 9	263·3	3·80	9·4	11·0	3	·756	
2427	+31·3800	46·2	31 43	64·2	4·01	9·3	11·0	2	·723	
2428	+31·3815	48·2	31 32	246·4	3·78	11·0	11·0	2	·801	BC
				92·2	87±	A=8·5		2	·801	AB

Espin No.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1930.	
	°	h m	° ′	°	″				
2429	..	19 48·6	31 41	112·1	5·35	9·5 12·7	4	+·818	
2430	+32·3668	56·8	32 15	270·9	5·57	9·5 12·0	2	·617	
2431	+31·3970	20 5·7	32 0	185·9	6·00	9·5 12·0	2	·652	
2432	..	13·0	31 46	114·9	5·31	10·0 10·0	2	·689	
2433	+31·4076	21·1	31 39	270·0	7·27	8·5 10·0	3	·656	
2434	+31·4074	21·1	31 21	286·1	4·76	10·0 10·2	2	·845	
2435	+30·4044	24·2	31 4	255·6	3·75	9·3 10·0	2	·862	
2436	+31·4120	28·0	31 16	316·0	5·40	9·3 10·5	3	·832	
2437	+31·4227	45·0	31 26	46·1	7·25	9·3 13·0	3	·826	
2438	+31·4284	53·6	31 38	311·3	6·95	9·5 12·5	2	·842	
2439	..	21 2·6	31 58	155·5	4·31	11·0 11·0	4	·874	
2440	+31·4356	5·6	31 19	202·4	5·00	9·5 10·5	2	·864	
2441	..	24·7	30 31	96·6	4·35	10·0 10·0	2	·689	
2442	..	25·1	30 23	255·6	5·40	10·0 11·0	3	·771	
2443	+36·5141	23 54·6	37 14	149·5	7·33	10·5 11·0	3	·989	BC
				232±	75±	A=7·0		·989	AB
2444	..	58·1	36 41	256·4	6·78	10·0 10·3	3	·955	

Notes.

2405. Companion very faint on all nights, and observations unsatisfactory.
2424. Two *comites* 13 and 13½ mags. at P. 289° and 225°.
2430. Third star, 11 mag. at P. 42°, D. 11½″.

Milburn No.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1930.
	°	h m	° ′	°	″			
631	..	0 6·1	+29 20	278·5	6·86	10·2 10·2	2	+·004 (1931)
632	..	12·6	30 2	270·3	1·99	10·0 11·0	2	·010 (1931)
633	..	44·3	29 25	16·9	5·97	9·7 10·7	2	·934
634	..	1 29·4	27 22	144·7	6·21	11·0 11·8	2	·059
635	..	49·9	27 3	165·9	6·63	9·4 9·8	2	·045
636	+28·371	2 7·5	28 24	177·2	4·03	9·3 14·0	3	·644
	+29·454	34·5	29 20	186·4	11·49	9·0 10·2	3	·994
637	+28·468	44·6	28 52	263·4	2·03	9·5 9·7	2	·045
638	..	3 14·7	27 57	299·2	8·08	9·6 10·2	2	·649
639	+27·611	54·0	27 16	267·0	7·31	8·4 10·0	2	·523
640	..	4 58·2	28 5	42·8	3·02	9·6 10·0	2	·059
641	+28·820	5 27·2	28 49	70·5	7·58	9·2 10·2	1	·066
642	..	40·4	28 46	140·4	4·13	9·7 11·2	2	·059

Milburn No.	B.D.	1900. R.A.	1900. Decl.	P.	D.	Mags.	Nts.	Date 1930.	
	°	h m	° ′	°	″				
643	..	5 40·7	28 51	59·2	4·56	9·7 12·2	2	+·059	
644	+28·1086	6 15·1	28 52	22·4	6·44	9·5 11·0	1	·066	
645	..	11 36·0	27 52	157·3	3·14	10·5 11·5	1	·328	
646	..	13 17·4	28 26	170·0	2·45	10·5 10·6	2	·346	
647	..	15 5·5	28 28	53·1	3·18	10·0 10·5	2	·409	
648	..	18 23·1	28 28	21·9	8·35	9·7 11·0	3	·628	
649	..	25·9	28 0	164·3	2·99	10·4 12·2	2	·634	
650	..	47·0	28 24	161·1	3·55	10·7 11·2	3	·606	
651	..	49·8	30 5	72·3	3·87	10·7 12·3	3	·722	
652	..	19 25·2	29 5	216·3	3·90	10·5 11·0	3	·606	
653	+28·3389	30·0	28 42	163·5	1·45	9·6 9·8	2	·667	
654	..	30·8	28 57	307·3	6·24	10·6 10·8	4	·762	
655	..	30·8	28 25	85·9	3·67	12·3 12·7	4	·687	
656	..	31·0	29 7	132·5	5·29	9·6 10·5	3	·819	
657	..	48·0	29 10	40·1	3·22	12·2 12·3	3	·756	
658	..	48·1	29 8	115·3	2·30	9·9 10·0	2	·736	
659	..	56·0	28 25	94·0	2·41	10·0 10·1	2	·617	
660	+28·3661	20 8·1	28 11	289·3	4·78	8·5 11·2	2	·845	
661	..	9·3	29 23	164·7	3·75	9·6 10·5	2	·642	
662	..	23·9	27 59	51·3	4·03	9·8 10·2	3	·849	
663	..	33·0	29 32	284·1	2·83	10·0 12·0	3	·756	
664	..	38·0	28 31	164·0	4·21	9·9 9·9	2	·675	
665	..	21 0·2	27 43	327·1	7·54	10·0 11·0	2	·850	
666	..	1·1	28 0	173·4	1·53	9·9 11·3	3	·861	
667	+29·4346	8·6	29 19	29·0	2·41	9·2 9·4	3	·879	
668	..	24·6	29 36	337·7	4·44	9·8 11·0	2	·913	
669	..	47·2	30 2	308·9	2·49	9·5 10·5	3	·820	
670	..	22 26·7	30 21	289·9	6·48	9·7 12·1	3	·884	AB
				230·9	7·71	C=11·7 3		·876	AC
671	+29·4687	28·3	29 50	355·5	4·86	9·5 10·0	3	·847	
672	..	23 35·2	29 26	110·5	4·06	9·9 12·3	2	·835	
673	+29·4989	40·9	30 6	291·8	5·48	9·0 10·1	2	·944	

Notes.

631. Star, 10·0 mag.; 38°·1; 40″±; 1931·003.
635. =A.G. Camb. 1045.
636. *Comes* very faint and observations unsatisfactory.
653. AC; 300°·3; 11″±; C=9·7; AD; 9°·2; 38″±; D=9·5, 1930·684.
671. B.D. star is a wide pair of equal stars, of which the *n* is the double.

B.D. Stars Missing or Faint.

		h m s	° ′				
(1)	+34°624	3 9 58·4	+34 56·5	Mag. 9·5	1930 Jan. 30.	Not seen.	E.
(2)	+28·518	11 59·4	+28 25·5	,, 9·5	Jan. 25.	Missing.	M.
(3)	+34·649	17 45·2	+34 56·1	,, 9·5	Jan. 30.	Not seen.	E.
(4)	+29·576	26 5·8	+29 25·7	,, 9·5	1931 Jan. 3, 6.	Missing.	M.
(5)	+35·801	57 59·6	+35 2·5	,, 9·5	1930 Jan. 20.	Not seen.	E.
(6)	+34·965	5 3 26·7	+34 42·5	,, 9·5	Jan. 20.	Not seen.	E.
(7)	+33·2069	10 55 12·8	+33 12·4	,, 9·5	May 1.	Not seen.	E.
(8)	+35·2374	12 41 14·7	+35 56·6	,, 9·5	May 1.	Not seen.	E.
(9)	+28·2990	18 18 3·0	+28 57·8	,, 9·5	Aug 7, 10, 14.	Not seen.	M.
(10)	+28·3596	19 56 54·3	+28 19·2	,, 9·5	Sept. 8.		M.
(11)	+28·3598	57 15·8	+28 18·0	,, 9·4	Sept. 8.		M.
(12)	+29·4988	23 37 53·5	+29 55·0	,, 9·5	Nov. 26.	11 mag.	M.

Notes.

No. 1. A 13½ mag. in Uccle chart +35° No. 22 in this place. E.
No. 2. Two faint stars, 11 mag. NF. this place. M.
Nos. 10 and 11. Given in Catalogue, but not in Chart. M.